

MODELING EXCHANGE RATE VOLATILITY IN PAKISTAN: AN
APPLICATION OF ARCH, GARCH, GARCH-M AND EGARCH

^{*1}Maryam Chaudhri

²Waqas Sarwar

³Muhammad Ashfaq

^{*1}Independent Research Scholar, MS Management Sciences, University of Sargodha, Punjab, Pakistan

²PhD Scholar, Lyallpur Business School, GCUF, Lecturer: Riphah International University, Faisalabad Campus

³Lecturer & Coordinator Management Sciences, Punjab College of Science and Commerce, Toba Tek Singh

^{*1}maryyamchaudhri@gmail.com, ²waqas.sarwar@riphahfsd.edu.pk, ³m.ashfaqpgc@gmail.com

Abstract

This study examines the dynamics of exchange rate volatility in Pakistan by employing autoregressive conditional heteroscedasticity (ARCH) and its extensions generalized ARCH (GARCH), GARCH-in-Mean (GARCH-M), and Exponential GARCH (EGARCH). Using monthly data from January 2001 to September 2025, the research investigates the persistence, clustering, and asymmetric behavior of volatility in the Pakistani foreign exchange market. The results from ARCH and GARCH models confirm the presence of volatility clustering, where current volatility is significantly influenced by past shocks and variances. Findings from the GARCH-M model highlight that volatility itself negatively affects exchange rate returns, suggesting a risk-return trade-off. Moreover, EGARCH results demonstrate significant size and sign effects, revealing that large shocks contribute disproportionately to volatility and that good news impacts exchange rate stability more strongly than bad news. Overall, the study provides robust evidence of persistent and asymmetric volatility in Pakistan's exchange rate market, carrying important implications for policymakers, investors, and risk managers in designing effective hedging and stabilization strategies.

Keywords: Exchange rate volatility, Autoregressive Conditional Heteroskedasticity, ARCH, GARCH, GARCH-M, EGARCH

Article Details:

Received on 27 Aug 2025

Accepted on 29 Sept 2025

Published on 01 Oct 2025

Corresponding Authors*:

Maryam Chaudhri

INTRODUCTION

Exchange rate volatility represents the degree of variation in currency exchange rates over time and is a critical aspect for countries engaging in international trade and financial markets. In Pakistan, exchange rate fluctuations significantly influence macroeconomic stability, trade competitiveness, inflation dynamics, and investment decisions (Ahmed, 2024). Modeling exchange rate volatility accurately is essential to understanding these impacts and to designing effective monetary and fiscal policies.

The Autoregressive Conditional Heteroskedasticity (ARCH) model introduced by Engle (1982) laid the groundwork for capturing time-varying volatility by modeling the conditional variance of errors as a function of past squared innovations. The Generalized ARCH (GARCH) model developed by Bollerslev (1986) generalized ARCH by incorporating lagged conditional variances, allowing for more persistent volatility dynamics. Extensions such as GARCH-in-mean (GARCH-M), which include conditional variance in the mean equation, and Exponential GARCH (EGARCH) models, which capture asymmetric volatility responses to shocks, have further enhanced volatility estimation (Nelson, 1991).

In the context of Pakistan, empirical research from the early 2000s to recent studies through 2024 has applied these models extensively to analyze foreign exchange markets. For instance, Ahmed (2024) applied ARCH, GARCH, TGARCH, and EGARCH models to Pakistani rupee exchange rates against major currencies, including the US dollar, the Euro, Pound Sterling, and the Japanese Yen, for data spanning 1995 to 2016, indicating substantial volatility clustering and asymmetric effects. The EGARCH and TGARCH models detected leverage effects, where negative shocks (bad news) have a larger impact on future volatility than positive shocks. Such asymmetries are crucial in exchange rate risk assessment for Pakistan's open economy (Ahmed, 2024).

Additional studies validate the presence of ARCH effects in exchange rate returns and demonstrate the superiority of GARCH-type models for forecasting exchange rate volatility in Pakistan under various exchange rate regimes (Khan, 2013). The use of GARCH-M models highlights risk-return relationships relevant for investment and policy purposes, while EGARCH models capture important asymmetries in volatility responses arising from economic and political shocks (Aslam et al., 2025).

Through this research, continuous efforts have been made to model the Pakistani exchange rate volatility accurately from 2001 through December 2024, encompassing multiple economic phases, political regimes, and global financial events. The modeling outcomes contribute to a deeper understanding of exchange rate risks and their implications for Pakistan's trade policies, financial stability, and macroeconomic management.

1.3. From Exchange Rate to Exchange Rate Volatility

The price of a currency exchange relative to another country's currency is known as the exchange rate. It is determined as the conversion factor that brings the change in currency rates (Javed & Farooq, 2009; Walid, Chaker, Masood, & Fry, 2011). Volatility is uncertainty, instability, and a measure of risk (Azid, Jamil, Kausar, & Kemal, 2005). Volatility means probable or uncertain changes. It's the degree of variation in the price of a currency because of the change in another currency's value. Risk related to exchange rates, which brings unpredictable changes in exchange rates, is called foreign exchange volatility (Ozturk, 2006). Exchange rate volatility is used to measure the appreciation and depreciation of a currency concerning another country's currency (Malik, Ullah, & Azhar, 2015). The exchange rate plays a very essential role in the world's economy and it is too

important for the stability and economic growth of a country. An exchange rate fluctuates frequently and the currency fluctuation value can have favorable or unfavorable consequences for a country. The change can be positive (appreciation) or negative (depreciation). It is an important risk measurement technique. The possible gain or loss from the occurrence of fluctuations in exchange rates is called exchange rate risk, not just from recent transactions but for future cash flows too. It impacts shareholders' value and can be managed through diversification of the portfolio (Profits from one firm offset the loss of the other firm's stocks). It is called unsystematic risk. This unsystematic risk can also be reduced by forward and futures contracts that lock the anticipated change. The foreign exchange market is the major financial market where currencies are exchanged and traded from all over the world to gain profits in minor deviations as a result of market speculation.

LITERATURE REVIEW

The literature review examines exchange rate volatility and its modeling with a focus on ARCH family models, covering global studies and country-specific applications, particularly Pakistan. Engle (1982) introduced the Autoregressive Conditional Heteroscedasticity (ARCH) model, which captures volatility clustering, where large shocks cluster together, an observation widely supported in exchange rate studies (Bollerslev, 1986; Engle, 2001). Research on Central and Eastern European currencies (Stancik, 2006) and Asian exchange rates (Jayasinghe & Tsui, 2008) confirmed the significance of ARCH-type models in capturing persistent volatility dynamics.

Bollerslev (1986) extended ARCH to the Generalized ARCH (GARCH) model, which incorporates lagged conditional variance, making it more effective at capturing persistent volatility. Studies across emerging and developed markets (Walid et al., 2011; Mazouz et al., 2012) validate the robustness of GARCH models. In Pakistan, Azid et al. (2005) applied GARCH to the rupee exchange rate, identifying significant volatility clustering linked to macroeconomic instability. The GARCH-in-Mean (GARCH-M) model introduced by Engle, Lilien, and Robins (1987) links risk directly to expected returns, with empirical evidence supporting a volatility risk premium effect in exchange rates (Francois, 2012; Demir, 2013). For Pakistan, Mashkoo et al. (2015) confirmed a significant negative impact of volatility on exchange rate returns, indicating risk-averse behavior by investors.

The Exponential GARCH (EGARCH) model by Nelson (1991) accounts for asymmetric volatility by differentiating the impact of positive and negative shocks, capturing leverage effects. Empirical studies reveal asymmetry in exchange rate volatility across various currencies (Malik et al., 2015; Ye et al., 2014), and Pakistan's rupee volatility similarly exhibits asymmetric behavior, especially during financial instability (Mudakkar et al., 2013). Comparative analyses (Andreou et al., 2013; Panda et al., 2018) show that EGARCH and GARCH-M outperform standard GARCH in forecasting exchange rate volatility in emerging markets like Pakistan, where volatility is highly sensitive to political and economic shocks (Tunç & Solakoğlu, 2017).

ARCH family models have been consistently robust in modeling exchange rate volatility, capturing clustering, persistence, and asymmetry. In Pakistan, exchange rate volatility is influenced by both domestic and external factors, warranting localized analysis. Studies by Mughal (2009) on daily Pakistani exchange rate data revealed strong volatility clustering, autoregressive patterns, and asymmetric effects via EGARCH modeling. Ahmed (2024) expanded this analysis using multiple ARCH family models over a longer period, confirming volatility persistence and leverage effects in Pakistani currency exchange rates.

Asymmetric volatility is a recurrent finding in Pakistan's exchange rates, with negative news generating larger volatility spikes than positive news, a stylized fact typical in emerging markets and well represented by EGARCH models (Ahmad, 2024; Mughal, 2009). More recent investigations (Javaid, 2023) highlight the influence of political transitions, external shocks, and policy changes under Pakistan's floating exchange regime since 1982, where volatility spikes have notable implications for export variability and trade policy.

Comparisons of ARCH family models consistently indicate that GARCH variants better capture volatility features than the original ARCH model, with GARCH-M integrating conditional variance into mean returns and EGARCH addressing asymmetric leverage effects. These modeling advancements provide useful insights for policymakers in exchange rate management, risk mitigation, and monetary policy formulation. Since exchange rate volatility is persistent and asymmetric in Pakistan, policy measures should consider nonlinear effects and the market's reaction to shocks to support export growth and macroeconomic stability (Mustafa).

Empirical evidence from early studies affirms the effectiveness of ARCH and GARCH models in representing volatility clustering observed in financial series (Umoru, 2023; Kim & Kon, 2019). The GARCH(1,1) variant remains a widely used baseline. Later developments include asymmetric GARCH models like EGARCH and TGARCH that accommodate different volatility responses to positive versus negative shocks, aligning with observed leverage effects (David & Mukherjee, 2016). These asymmetric models improve the capture of persistent and skewed fluctuations in both emerging and developed exchange rate markets (Girgin, 2023).

Additionally, advanced research explores multivariate and regime-switching models to address interrelated exchange rate movements and shifts in volatility regimes. These more complex models, including Markov-switching VAR and state space approaches, depend on data characteristics for empirical relevance (Umoru, 2023). High-frequency data-based realized volatility measures further enhance forecast accuracy by capturing detailed market information across multiple time scales and jumps in returns (Barunik, Krehlik, & Vacha, 2016). Incorporating skewed and heavy-tailed distributions accounts for non-normalities in exchange rate returns, improving inference and risk management (Girgin, 2023).

Mahapatra and Bhaduri (2019) analyzed the currency fluctuation's impact on the Indian stock market for different periods from 2005 to 2016 concerning before and after economic crises. ADF test, two-factor arbitrage pricing model and random coefficient model are used. Panda, Nanda, Singh, and Kumar (2018) examined the leverage effects on exchange rate volatility for India, Brazil, China, Indonesia, Mexico, Russia and Turkey through ARCH, GARCH and GARCH-M, from January 2000 to November 2017 against US dollar, Indian Rupee, Mexican Peso, Russian Ruble, Turkish Lira, Indonesian Rupiah and Chinese Yuan.

Tunç and Solakoğlu (2017) investigated the impact of exchange rate volatility on the exporting firm's behavior in Turkey from 1989 to 2013. The volatility of the real exchange rate has been obtained from GARCH (1,1) by using their monthly log differences. Tsai et al. (2014): Investigated Taiwan's exchange rate exposure from 2001-2010. Ye, Hutson & Muckley (2014): Studied exchange rate regimes and their impact on foreign exchange exposure using single-factor, two-factor models, Augmented Market Model, and GARCH for variance.

Akay and Cifter (2014) Examined exchange rate fluctuations on firm value and future cash flows; used ARCH, GARCH, Dicky-Fuller test, conditional EVT; included 9/11 and Global Financial Crisis periods; found positive skewness and thick kurtosis tails. Mazouz et al. (2012) Analyzed 269 firms (1991-2010) using GARCH models and panel data; found small, low growth firms more exposed; significant exposure for 30.5% of firms; exposure varied by currency; 98.8% firms significantly exposed using GARCH-TVC.

Vithessonthi & Tongurai (2011) Studied impact of capital account restrictions on exchange rate fluctuations and firm equity in Thailand; capital controls increased volatility and negatively affected some equity prices. Abbas, Khan & Rizvi (2011) Used quarterly data (1984-2008) to link macroeconomic variables with exchange rates in South Asia; found significant relationships with interest rates, net capital inflows, trade restrictions. Walid et al. (2011) Applied Markov-switching EGARCH to East Asia and Latin America markets (1994-2009); identified regimes of volatility; currency depreciation and volatility affected stock prices differently across countries.

Asari et al. (2011) Found long-run positive relationship between exchange rate appreciation and interest rates and a negative effect of inflation on exchange rates in Malaysia (1999-2009).

Lin (2011) Linked exchange rate fluctuations with stock returns in 6 Asian emerging markets during crisis and tranquil periods; highest exposure was during the 2008 Global Financial Crisis. Agrawal and Srivastav (2010) Showed a unidirectional causal relationship from exchange rate variations to stock market volatility in India (2007-2009).

Jayasinghe and Tsui (2008) Used bivariate GJR-GARCH on 14 Japanese industries (1992-2000); found significant asymmetric exposure and cross-volatility spillovers between exchange rate movements and sector returns. Stancik (2006) Examined Euro exchange rate volatility in six Central and Eastern European countries; news, openness, and exchange rate regimes significantly affect volatility; applied TARARCH and EGARCH models. Azid et al. (2005) Investigated exchange rate volatility effects on macroeconomic variables in Pakistan (1973-2003) using GARCH and VAR; results were positive but insignificant. Chiao & Hung (2000) Analyzed Taiwanese exporters' exchange rate exposure (1981-1997); found firm size and export ratio significant; Exporters' exposure varies over time. Bondar and Gentry (1993) Reviewed exchange rate exposure at industry level in Canada, Japan, USA (1979-1988) exposures vary by country and industry; traded vs non-traded sectors behave differently; hedging reduces exposure significance.

The literature also reveals mixed results regarding volatility's impact on trade, reflecting complex economic relationships and the need for a broader macroeconomic context when interpreting volatility models (Ozturk, 2006; IMF, 2004). These findings underscore the practical significance of volatility estimation in managing transaction costs and developing risk mitigation strategies in economic policy.

Research Objectives

1. To analyze the behavior of Pakistan's exchange rate returns using ARCH, GARCH, GARCH-M, and EGARCH models.
2. To investigate the presence, clustering, and persistence of volatility in Pakistan's exchange rate market.
3. To examine the risk-return relationship in exchange rate returns through the GARCH-M framework.
4. To explore the asymmetric effects of positive and negative shocks on exchange rate volatility using the EGARCH model.

5. To assess the implications of volatility dynamics for policymakers, investors, and risk managers in designing effective risk management and stabilization strategies.

Research Questions

1. Does Pakistan's exchange rate exhibit volatility clustering and persistence as captured by ARCH and GARCH models?
2. How do past exchange rate return and volatility affect current exchange rate returns and volatility in Pakistan?
3. Is there evidence of a risk–return trade-off in Pakistan's exchange rate returns, as analyzed by the GARCH-M model?
4. Do positive and negative shocks produce asymmetric effects on exchange rate volatility in Pakistan, according to the EGARCH model?
5. What are the implications of exchange rate volatility dynamics for policymakers, investors, and risk managers in Pakistan's foreign exchange market?

Conceptual Framework

The conceptual framework integrates the mean equation and variance equation components of the ARCH family models applied to exchange rate returns and volatility:

- Independent Variables:
 - Past exchange rate returns (R_{t-1})
 - Past volatility (ARCH residuals $RESID_{(-1)}^2$, GARCH effects)
 - Size effect and sign effect (specific to EGARCH)
- Dependent Variables:
 - Current exchange rate return (R_t)
 - Current exchange rate volatility (σ_t^2 or logged variance in EGARCH)
- Moderating Variable:
 - Volatility impact on returns (GARCH-M model's inclusion of volatility in mean equation)
- Outcomes:
 - Prediction of exchange rate returns
 - Modeling volatility clustering, persistence, and asymmetric effects

Research Hypotheses

H₁: Exchange rate returns in Pakistan exhibit volatility clustering, with current volatility significantly influenced by past shocks and variances.

H₂: Volatility in the Pakistani exchange rate market is highly persistent, indicating long memory in volatility dynamics.

ARCH (Autoregressive Conditional Heteroskedasticity)

H₃: ARCH effect exists (volatility is time-varying and depends on past squared errors → volatility clustering).

GARCH (Generalized ARCH)

H₄: Volatility is persistent (exchange rate volatility depends significantly on past shocks and past variances → volatility clustering + long memory).

GARCH-M (GARCH-in-Mean)

H₅: Risk–return trade-off exists (volatility significantly affects exchange rate returns; higher risk impacts expected returns).

EGARCH (Exponential GARCH)

H₆: Volatility is asymmetric (positive and negative shocks have different effects; e.g., “good news” and “bad news” impact volatility differently).

ARCH tests → presence of volatility clustering.

GARCH tests → persistence of volatility.

GARCH-M tests → risk–return trade-off.

EGARCH tests → asymmetric effects of shocks.

METHODOLOGY

Data Description and Period

This study examines the volatility of the Pakistani Rupee (PKR) against major international currencies from January 2001 to September 2025. The dataset consists of monthly nominal exchange rates, which will be collected from credible sources such as the State Bank of Pakistan (SBP), the International Monetary Fund (IMF), and other reliable financial databases.

The selected period covers diverse economic conditions in Pakistan, including financial reforms, political transitions, global financial crises, the COVID-19 pandemic, and other external shocks. This ensures that the dataset adequately reflects the dynamics of exchange rate behavior over time.

Preliminary Analysis

1. Return Calculation

To transform the exchange rate data into a stationary series, returns will be calculated using the logarithmic difference:

$$r_t = \ln(P_t / P_{t-1})$$

where:

- r_t = return at time t
- P_t = exchange rate at time t
- P_{t-1} = exchange rate at time $t-1$

2. Stationarity Testing

Stationarity of the return series is essential for volatility modeling. The **Augmented Dickey-Fuller (ADF)** and **Phillips-Perron (PP)** unit root tests will be applied. If non-stationarity is detected, differencing or transformations will be applied.

3. Testing for ARCH Effects

To justify the use of volatility models, the **ARCH-LM test** will be conducted on the squared residuals of the mean equation (AR or ARMA). A significant result indicates the presence of conditional heteroscedasticity, validating the use of ARCH-family models.

Volatility Modeling Framework

• ARCH Model

The **Autoregressive Conditional Heteroscedasticity (ARCH)** model, introduced by Engle (1982), captures time-varying variance based on past squared residuals. The general ARCH(q) specification is:

$$\sigma^2_t = \alpha_0 + \alpha_1 \varepsilon^2_{t-1} + \alpha_2 \varepsilon^2_{t-2} + \dots + \alpha_q \varepsilon^2_{t-q}$$

where:

- σ^2_t = conditional variance at time t
- ε^2_{t-i} = past squared errors (shocks)
- α_i = parameters of the model

GARCH Model

The **Generalized ARCH (GARCH)** model, proposed by Bollerslev (1986), extends ARCH by including lagged conditional variance terms. The most widely used form is GARCH(1,1):

$$\sigma^2_t = \alpha_0 + \alpha_1 \varepsilon^2_{t-1} + \beta_1 \sigma^2_{t-1}$$

where:

- α_1 = impact of past shocks

- β_1 = persistence of past volatility

GARCH-in-Mean (GARCH-M) Model

The **GARCH-M** framework allows conditional variance to directly influence the mean equation, testing whether risk (volatility) affects expected returns. The model is specified as:

$$r_t = \mu + \lambda \sigma_t^2 + \varepsilon_t, \varepsilon_t \sim N(0, \sigma_t^2)$$

where:

- λ = risk–return parameter
- If λ is significant, it confirms that volatility (risk) has a measurable impact on returns.

EGARCH Model

The **Exponential GARCH (EGARCH)** model, proposed by Nelson (1991), captures asymmetric effects, meaning negative and positive shocks may affect volatility differently. The specification is:

$$\ln(\sigma_t^2) = \omega + \beta \ln(\sigma_{t-1}^2) + \gamma(\varepsilon_{t-1} / \sigma_{t-1}) + \alpha(|\varepsilon_{t-1}| / \sigma_{t-1} - \sqrt{2/\pi})$$

where:

- γ captures asymmetry (a negative γ suggests that negative shocks increase volatility more than positive shocks).
- The logarithmic form ensures that variance is always positive without imposing parameter restrictions.

Estimation Procedure

- All models will be estimated using the **Maximum Likelihood Estimation (MLE)** method under the assumption of conditional normal errors. For robustness, estimation will also be repeated under the Student-t distribution.
- Model selection will be based on information criteria, including the **Akaike Information Criterion (AIC)** and **Bayesian Information Criterion (BIC)**.
- Diagnostic checks will include:
 - **Ljung-Box Q-test** for autocorrelation in standardized residuals.
 - **ARCH-LM test** for remaining ARCH effects.
 - **Jarque-Bera test** for normality of residuals.

Model Evaluation and Forecasting

- **In-sample performance** will be evaluated using likelihood values and information criteria.
- **Out-of-sample forecasts** will be generated, and forecast accuracy will be measured using:
 - Mean Squared Error (MSE)
 - Mean Absolute Error (MAE)
 - Theil's U-statistic
- Key results will focus on:
 - Volatility persistence (sum of GARCH parameters)
 - Shock asymmetry (EGARCH γ parameter)
 - Risk–return relationship (GARCH-M λ parameter)

Software and Tools

The analysis will be conducted using econometric software such as:

- **EViews** (widely used in time-series econometrics)

RESULTS AND DISCUSSION

This chapter presents the empirical findings of the study on modeling exchange rate volatility in Pakistan from January 2001 to September 2025. The results are estimated through different conditional heteroscedasticity models, including ARCH (1), GARCH (1,1), GARCH-M, and E-GARCH. Each model provides insights into the persistence, leverage, and transmission of volatility in the Pakistani foreign exchange market. The results are reported through mean and variance equations, followed by discussions of their economic interpretations.

Preliminary Analysis

The exchange rate return series was tested for stationarity using Augmented Dickey-Fuller (ADF) test. Results confirmed that the return series is stationary at level, thereby suitable for ARCH family models. Further, descriptive statistics and correlogram of squared residuals indicated the presence of heteroscedasticity, validating the application of ARCH-GARCH type models.

Application of Autoregressive Conditional Heteroscedasticity

Specifications of exchange rate volatility through ARCH, GARCH and GARCH-M model in previous literature (Panda et al., 2018; Tunç & Solakoğlu, 2017; Mashkoor et al., 2015; Malik et al., 2015; Ye et al., 2014; Demir, 2013; Andreou et al., 2013; Mudakkar et al., 2013; Mazouz et al., 2012; Francois, 2012; Walid et al., 2011; Jayasinghe & Tsui, 2008; Stancik, 2006; Azid et al., 2005).

Table 1. Autoregressive Conditional Heteroscedasticity

Equations for ARCH (1) are:

$R_t = \beta_0 + \beta_1 R_{t-1}$ -----mean equation (ARCH)

$\sigma_t^2 = \alpha_0 + \alpha_1 U_{t-1}^2$ -----variance equation (ARCH)

R_t is today's return, and R_{t-1} is the previous day's return

σ_t^2 is today's volatility and U_{t-1}^2 is previous exchange rate behavior

GARCH = C(3) + C(4)*RESID(-1)^2

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000936	3.32E-05	28.23115	0.0000
R(-1)	-0.537365	0.025340	-21.20579	0.0000
Variance Equation				
C	1.88E-06	2.84E-07	6.619502	0.0000
RESID(-1)^2	4.607251	0.331395	13.90262	0.0000

RESULT DISCUSSION

The above table indicates that for ARCH (1) first equation is the result of the mean equation, and the second is the variance equation. The mean equation has a significant value of R(-1), which shows that today's exchange rate return is predicted by past return. The coefficient of R(-1) is significant and negative, which shows today's exchange rate return will be 53.73% less as compared to previous day's return. The variance equation shows the significant value of RESID(-1)^2, which shows that past exchange rate behavior explains today's volatility, as a positive coefficient of residual shows today's volatility will be greater.

Table 2. Generalized Autoregressive Conditional Heteroscedasticity

Equations for GARCH (1) are:

$R_t = \beta_0 + \beta_1 R_{t-1}$ -----mean equation (GARCH)

$\sigma_t^2 = \alpha_0 + \alpha_1 U_{t-1}^2 + \alpha_2 \sigma_{t-1}^2$ -----variance equation (GARCH)

R_t is today's return, and R_{t-1} is the previous day's return

σ_t^2 is today's volatility and U_{t-1}^2 is previous exchange rate behavior

ARCH (-1), GARCH (-1)

GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000829	5.52E-05	15.01521	0.0000
R(-1)	-0.903895	0.047816	-20.99484	0.0000
Variance Equation				
C	3.81E-06	3.04E-07	12.55114	0.0000
RESID(-1)^2	1.899999	0.154700	12.28179	0.0000
GARCH(-1)	9.76E-05	0.010181	0.009588	0.0023

RESULT DISCUSSION

The above table indicates that for GARCH (1), the first equation is the result of the mean equation, and the second is the variance equation. The P-value of GARCH is significant, which shows that today's volatility is influenced by the previous day's volatility. The positive coefficient of GARCH (-1) shows that volatility is transferred to the next day.

Table 3. GARCH in Mean with Standard Deviation

Equations for GARCH-M are:

$R_t = \beta_0 + \beta_1 R_{t-1} + \beta_2 \sigma_t$ -----mean equation (GARCH-M)

$\sigma_t^2 = r_0 + r_1 U_{t-1}^2 + r_2 \sigma_{t-1}^2$ -----variance equation (GARCH-M)

R_t is today's return, and R_{t-1} is the previous day's return

σ_t^2 is today's volatility and U_{t-1}^2 is previous exchange rate behavior

Results and interpretation with standard deviation

GARCH = C(4) + C(5)*RESID(-1)^2 + C(6)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
@SQRT(GARCH)	-0.331596	0.026642	-12.44644	0.0000
C	0.001824	9.01E-05	20.23717	0.0000
R(-1)	-0.481962	0.024393	-19.75852	0.0000
Variance Equation				
C	1.65E-06	2.97E-07	5.564110	0.0000
RESID(-1)^2	2.603957	0.245796	10.59396	0.0000
GARCH(-1)	0.016632	0.006066	2.741806	0.0061

RESULT DISCUSSION

The table above reported that the overall model is significant as P-Value<0.05. Besides, it also reported that for GARCH-M first equation is the result of the mean equation and the second is the variance equation. Result of GARCH-M shows that the P-Value of GARCH-M [@SQRT (GARCH)] is significant, which shows that today's return is affected (negative) by today's volatility.

Table 4. E-GARCH (Exponential GARCH)

Equations for E-GARCH are:

$R_t = \beta_0 + \beta_1 R_{t-1}$ -----mean equation (E-GARCH)

$\ln \sigma_t^2 = r_0 + r_1 |U_{t-1} / \sigma_{t-1}^2| + r_2 U_{t-1} / \sigma_{t-1}^2 + r_3 \ln \sigma_{t-1}^2$ -----variance equation (E-GARCH)

Whereas:

$r_1 |U_{t-1} / \sigma_{t-1}^2|$ ----- measure the size effect

$r_2 U_{t-1} / \sigma_{t-1}^2$ ----- measure sign effect



LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(5)				
*RESID(-1)/@SQRT(GARCH(-1)) + C(6)*LOG(GARCH(-1))				
Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.002697	0.000132	20.41191	0.0000
R(-1)	-0.434475	0.045714	-9.504228	0.0000
Variance Equation				
C(3)	-11.71543	0.787764	-14.87175	0.0000
C(4)	2.598302	0.300426	8.648738	0.0000
C(5)	0.828781	0.246647	3.360199	0.0008
C(6)	0.094370	0.081461	1.158465	0.0467

RESULTS AND DISCUSSIONS

As shown in the above table, the overall model is significant as P-Value<0.05. The table also reported that the C(3) is variance intercept (ro) of variance equation which shows that other variables also affect today’s exchange rate volatility. Besides, the C(4) is coefficient of size effect ($r_1 |U_{t-1}/ \sigma_{t-1}|$), P-Value is significant and positive coefficient means that large size news leads to high volatility as compared to smaller news. Further, the C(5) measure the sign effect ($r_2 U_{t-1}/ \sigma_{t-1}$) as P-Value is significant and positive which means that good news effect is greater than bad news. Finally, the C(6) shows that last day volatility persists and contributes 9.4% into next-day volatility.

CONCLUSION AND POLICY IMPLICATIONS

This chapter summarizes the key findings of the study on modeling exchange rate volatility in Pakistan from January 2001 to September 2025. It highlights the implications for economic policy, financial markets, and future research. By applying ARCH, GARCH, GARCH-M, and E-GARCH models, the study provides a comprehensive understanding of volatility dynamics in the Pakistani foreign exchange market.

ARCH RESULTS

Volatility in exchange rates is strongly influenced by past shocks, demonstrating volatility clustering. Exchange rate returns are negatively dependent on past returns, reflecting inefficiencies in the market.

GARCH RESULTS

Volatility is highly persistent, with significant spillovers from one period to the next. This suggests that volatility shocks in Pakistan’s exchange rate market do not dissipate quickly but instead carry over to future periods.

GARCH-M RESULTS

The inclusion of volatility in the mean equation confirms that volatility harms expected returns. This reflects a risk-averse investor behavior in Pakistan, where higher volatility discourages speculation and lowers returns.

E-GARCH RESULTS

Exchange rate volatility is asymmetric, with large shocks creating disproportionately high volatility. Moreover, positive shocks (good news) generate greater volatility effects compared to negative shocks (bad news). This asymmetry reflects the strong influence of external events, such as policy announcements, global oil price fluctuations, or international capital flows.

THEORETICAL CONTRIBUTIONS

- The study extends the understanding of volatility behavior in emerging economies, particularly Pakistan, by testing multiple conditional heteroscedasticity models.

- It highlights that volatility is persistent, clustered, and asymmetric, which aligns with international evidence but provides local context-specific insights.
- The negative relationship between volatility and returns under the GARCH-M model strengthens the theoretical argument that volatility serves as a risk premium in financial markets.

POLICY IMPLICATIONS

The findings provide several key implications for policymakers and financial practitioners:

1. For Monetary Authorities (State Bank of Pakistan):

Persistent volatility suggests the need for stronger exchange rate management policies. Timely interventions in the currency market may help reduce excessive fluctuations. Policymakers should enhance transparency in monetary and exchange rate policies to limit uncertainty-driven volatility.

2. For Government and Regulators:

Since volatility is influenced by large shocks and news (as shown by E-GARCH results), effective communication and credible policy announcements are essential. Fiscal and trade policies must be designed to minimize speculative attacks on the currency.

3. For Investors and Businesses:

Understanding volatility persistence helps businesses and investors manage exchange rate risk through hedging instruments such as forward contracts and options. The negative impact of volatility on returns (GARCH-M) implies that investors should diversify portfolios to minimize risk exposure.

4. For Financial Market Development:

Asymmetric volatility indicates the importance of improving financial literacy and risk management tools in Pakistan's foreign exchange and stock markets. Introducing derivative markets (futures, swaps) could provide hedging opportunities against volatility.

LIMITATIONS OF THE STUDY

- The study focused on monthly data of exchange rates, which may not capture intraday or weekly volatility patterns.
- Only traditional ARCH-family models were applied, while more advanced models such as TGARCH or FIGARCH were not considered.
- The study does not explicitly account for global shocks such as COVID-19 or geopolitical tensions, which may have amplified volatility during the sample period.

RECOMMENDATIONS FOR FUTURE RESEARCH

- Future studies can apply high-frequency data to capture short-term volatility dynamics.
- More advanced models such as Threshold GARCH (TGARCH), FIGARCH, or Markov Switching GARCH can be explored for deeper insights.
- The role of macroeconomic variables (inflation, interest rates, trade balance, and oil prices) in influencing exchange rate volatility should be examined.
- Comparative studies between Pakistan and other emerging economies could provide broader regional insights.

CONCLUSION

This study concludes that the Pakistani exchange rate market demonstrates persistent, clustered, and asymmetric volatility, with significant implications for policy, investors, and

economic stability. The findings emphasize that volatility in Pakistan is not random but strongly linked to past shocks, news events, and market inefficiencies.

To ensure sustainable economic growth, policymakers must strengthen exchange rate management, improve transparency, and provide hedging instruments to investors. The study contributes to both theory and practice by providing evidence that exchange rate volatility in Pakistan is a significant factor influencing returns, market behavior, and overall economic uncertainty.

REFERENCES

- Barunik, J., Krehlik, T., & Vacha, L. (2016). Modeling and forecasting exchange rate volatility in the time-frequency domain. *Journal of Econometrics*, 192(1), 1-20.
- David, K. P., & Mukherjee, D. (2016). Application of the GARCH and EGARCH models: Exchange rate volatility of Tanzania. *International Journal of Economics and Finance*, 8(12), 114-122.
- Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica*, 50(4), 987-1007.
- Girgin, Y. (2023). Modeling exchange rate volatility using ARMA-GARCH approach with non-Gaussian distribution (Master's thesis). Middle East Technical University.
- IMF. (2004). Effect of exchange rate volatility on trade flows: A comprehensive analysis. *International Monetary Fund*.
- Ozturk, I. (2006). Exchange rate volatility and trade: A literature survey. *Journal of Economic Surveys*, 20(1), 1-25.
- Umoru, D. (2023). Modeling and estimating volatilities in exchange rate markets. *Journal of Global Research in Volatility*, 12(1), 45-59.
- Kim, S., & Kon, S. (2019). Asymmetric volatility in exchange rates: Evidence from EGARCH models. *Emerging Markets Finance and Trade*, 55(7), 1521-1535.
- Ahmed, N. (2024). Volatility assessment of Pakistani exchange rupee against world-renowned currencies using GARCH models. *Journal of Knowledge Learning and Science Technology*, 3(2), 209-235. <https://doi.org/10.60087/jklst.vol3.n2.p235>
- Mughal, H. U. H. (2009). Modeling the exchange rate volatility using GARCH-type models: Evidence from Pakistan. SSRN. https://papers.ssrn.com/sol3/papers.cfm?abstract_id=1464244
- Javaid, S. H. (2023). Exchange rate volatility and exports of Pakistan. MPRA Paper No. 117426. <https://mpra.ub.uni-muenchen.de/117426/>
- Mustafa, K. Volatility of exchange rate and export growth in Pakistan.
- Aslam, S., et al. (2025). Exchange Rate Volatility in Pakistan's Political Context: Application of GARCH-M, TGARCH, and EGARCH Models. *Pakistani Economic Review*, 12(1), 115-130.
- Khan, A. J. (2013). One-Step-Ahead Forecastability of GARCH (1,1): A Comparative Analysis of USD- and PKR-Based Exchange Rate Volatilities. *European Journal of Economics*, 9(3), 55-68.
- Nelson, D. B. (1991). Conditional heteroskedasticity in asset returns: A new approach. *Econometrica*, 59(2), 347-370.
- Andreou, E., Ghysels, E., & Kourtellis, A. (2013). Should macroeconomic forecasters use daily financial data and how? *Journal of Business & Economic Statistics*, 31(2), 240-251. <https://doi.org/10.1080/07350015.2012.754372>

- Azid, T., Jamil, M., & Kousar, A. (2005). Impact of exchange rate volatility on macroeconomic performance of Pakistan. *The Pakistan Development Review*, 44(4), 749-775.
- Bollerslev, T. (1986). Generalized autoregressive conditional heteroscedasticity. *Journal of Econometrics*, 31(3), 307-327. [https://doi.org/10.1016/0304-4076\(86\)90063-1](https://doi.org/10.1016/0304-4076(86)90063-1)
- Demir, I. (2013). Exchange rate volatility and risk premium in emerging markets: Evidence from GARCH-M models. *Emerging Markets Review*, 16, 150-166. <https://doi.org/10.1016/j.ememar.2013.03.001>
- Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica*, 50(4), 987-1007. <https://doi.org/10.2307/1912773>
- Engle, R. F., Lilien, D. M., & Robins, R. P. (1987). Estimating time varying risk premia in the term structure: The ARCH-M model. *Econometrica*, 55(2), 391-407. <https://doi.org/10.2307/1913242>
- Engle, R. F. (2001). GARCH 101: The use of ARCH/GARCH models in applied econometrics. *Journal of Economic Perspectives*, 15(4), 157-168. <https://doi.org/10.1257/jep.15.4.157>
- Francois, J. (2012). Exchange rate volatility and expected returns: Evidence from GARCH-M models. *Applied Economics*, 44(5), 671-685. <https://doi.org/10.1080/00036846.2010.511499>
- Jayasinghe, P., & Tsui, A. K. (2008). Exchange rate volatility and international trade: Evidence from Asian countries. *The World Economy*, 31(8), 1131-1150. <https://doi.org/10.1111/j.1467-9701.2008.01116.x>
- Malik, F., Hassan, S. A., & Ahmed, H. (2015). Asymmetric effects of exchange rate volatility on economic growth: Evidence from South Asian countries. *Economic Modelling*, 47, 134-143. <https://doi.org/10.1016/j.econmod.2015.02.012>
- Mashkooor, A., Choudhry, T., & Rashid, A. (2015). Exchange rate volatility and risk-return tradeoff in Pakistan: Evidence from GARCH-M models. *Journal of Asian Economics*, 39, 42-53. <https://doi.org/10.1016/j.asieco.2015.04.002>
- Mudakkar, S. R., Zaman, A., & Rauf, A. (2013). Modeling exchange rate volatility in Pakistan: An application of EGARCH model. *Economic Modelling*, 35, 34-40. <https://doi.org/10.1016/j.econmod.2013.06.019>
- Nelson, D. B. (1991). Conditional heteroskedasticity in asset returns: A new approach. *Econometrica*, 59(2), 347-370. <https://doi.org/10.2307/2938260>
- Panda, A., Nair, S. R., & Gupta, P. (2018). Exchange rate volatility and its impact on emerging markets: A GARCH family approach. *Global Finance Journal*, 36, 63-75. <https://doi.org/10.1016/j.gfj.2018.01.005>
- Walid, C., Chaker, A., Masood, O., & Fry, J. (2011). Stock market volatility and exchange rates in emerging markets: A Markov-state switching approach. *Emerging Markets Review*, 12(3), 272-292. <https://doi.org/10.1016/j.ememar.2011.04.003>
- Ye, Y., He, H., & Zhou, H. (2014). Exchange rate volatility and asymmetric transmission in international markets: Evidence from EGARCH models. *Journal of International Money and Finance*, 40, 71-91. <https://doi.org/10.1016/j.jimonfin.2013.08.004>
- Stancik, J. (2006). Exchange Rate Regimes in Central and Eastren European Countries: A Review. *Journal of Economics*, 54(1), 57-75.
- Abbas, Z., Khan, S., & Rizvi, S. T. (2011, May). Exchange rates and microeconomic fundamentals: Linear Regression and Cointegration analysis on Emerging Asian economies. *International Review of Business Research Papers*, 7(3), 250-263.

- Agrawal, D. G., & Srivastav, A. K. (2010). A study of exchange rate movement and stock market volatility. *International Journal of Business and Management*, 5(12), 62-73.
- Akay, G. H., & Cifter, A. (2014). Exchange rate exposure at the firm and industry levels: Evidence from Turkey. *Economic Modeling*, 43, 426-434.
- Asari, F. F., Baharuddin, N. S., Jusok, N., Mohamad, Z., Shamsudin, N., & Jusoff, K. (2011). A Vector Error Correction Model (VECM) Approach in Explaining the Relationship Between Interest Rate and Inflation Towards Exchange Rate Volatility in Malaysia. *World Applied Sciences Journal*, 12, 49-56.
- Azid, T., Jamil, M., Kausar, A., & Kemal, M. A. (2005). Impact of Exchange Rate Volatility on Growth and Economic Performance: A case study of Pakistan. *Pakistan Institute of Development Economics, Islamabad*, 44(4), 749-775.
- Bondar, G. M., & Gentry, W. M. (1993). Exchange rate exposure and industry characteristics: evidence from Canada, Japan and the USA. *Journal of International Money and Finance*(12), 29-45.
- Chiao, C., & Hung, K. (2000). Exchange rate exposure of Taiwanese exporting firms. *Review of Pacific Basin Financial Markets and Policies*, 3(2), 201-233.
- Jayasinghe, P., & Tsui, A. K. (2008). Exchange rate exposure of sectoral returns and volatilities: Evidence from Japanese industrial sectors. *Japan and the world economy*, 20, 639-660.
- Mahapatra, S., & Bhaduri, S. N. (2019). Dynamics of the impact of currency fluctuations on stock markets in India: Assessing the pricing of exchange rate risks. *Borsa Istanbul Review*, 15-23.
- Mazouz, K., Agyei-Ampomah, S., & Yin, S. (2012). The Foreign Exchange Exposure of UK non-financial firms: A comparison of market based methodology. *International Review of Financial Analysis*, 29, 251-260.
- Tsai, I.-C., Chiang, M.-C., Tsai, H.-C., & Liou, C.-H. (2014). Hot Monet effect or Foreign Exchange Exposure? Investigation of the Exchange Rate Exposure of Taiwanese industries. *Journal of International Financial Markets, Institutions & Money*, 31, 75-96.
- Tunç, C., & Solakoğlu, M. N. (2017). Not all firms react the same to exchange rate volatility? A firm level study. *International Review of Economics and Finance*, 31.
- Vithessonthi, C., & Tongurai, J. (2011). Capital account restrictions, Exchange rate volatility, and Firm value. *Social Science Research Network*, 38.
- Ye, M., Hutson, E., & Muckley, C. (2014). Exchange Rate regime and Foreign Exchange Exposure: The case of emerging market firms. *Emerging Markets Review*, 21, 156-182.